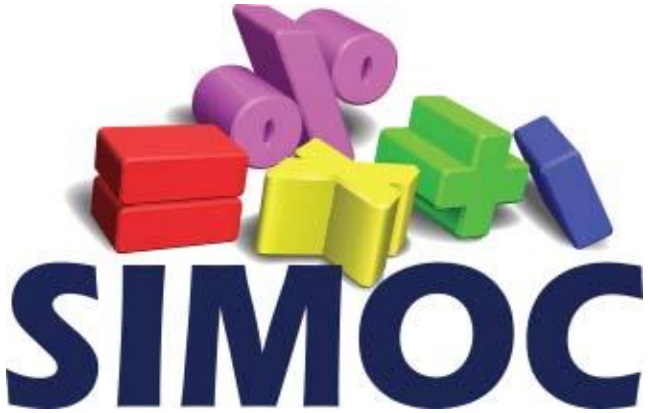
Teacher Copy

Middle Division
Grade 5, 6 & 7

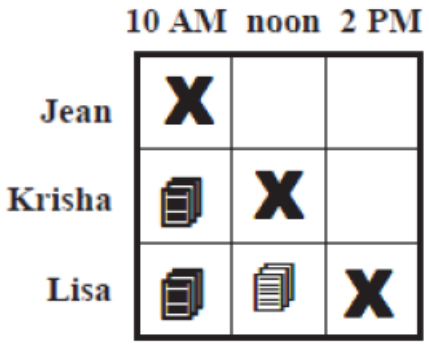


Logic & Non-Routine Problems

Example: Each of three Math Team students practices at a different time: 10:00 AM, noon, and 2:00 PM. Jean practices either at noon or at 2:00 PM. Krisha doesn't practice at noon, Lisa doesn't practice at 2:00 PM, and Krisha practices 2 hours before Lisa. At what time does each girl practice?

Solution: Mark an X each time that each girl does not practice.

(1) Jean does not practice at 10 AM.
(2) Krisha does not practice at noon.
(3) Lisa does not practice at 2 PM.
(4) Since Krisha practices 2 hours before Lisa, mark in the appropriate boxes to denote that Krisha must practice at 10 AM and Lisa at noon. Then Jean practices at 2 PM.



Notice that when the table is completed, will appear just once in each row and just once in each column.

Answer: **2 pm**

Logic & Non-Routine Problems

1. Joshua, Nicholas and Dickson are old classmates. During a recent gathering, they told each other about their occupation. Among them, there is a lawyer, a teacher and a manager. Below are the hints about their occupations:

Dickson is older than the lawyer.
Joshua's age is not the same as the teacher.
The teacher is younger than Nicholas.

What is their occupation?

Logic & Non-Routine Problems

Solution:

Case 1: Joshua is manager, Nicholas is lawyer,

Dickson is older than Nicholas; Joshua's age is not the same as Dickson, Dickson is younger than Nicholas. This leads to a contradiction. So, it cannot be the solution.

	Lawyer	Teacher	Manager
Dickson	X	✓	
Joshua		X	
Nicholas		X	

Case 2: Joshua is lawyer, Nicholas is manager,

Dickson is older than Joshua; Joshua's age is not the same as Dickson; Dickson is younger than Nicholas.

Oldest		Youngest
Nicholas	Dickson	Joshua

Answer: Joshua-lawyer, Nicholas-manager, Dickson-teacher

Logic & Non-Routine Problems

2. There are 100 Primary 5 students in Happy Primary School, 78 of these students are in Swimming club and 86 are in Badminton club. What is the largest possible number of students who are neither in Swimming club nor Badminton club?

Solution: To have largest possible number of students not in Swimming and Badminton club, we need to have smallest possible number of students who are in either Swimming or Badminton club. The smallest possible number is 86, where all 78 students are in both Swimming and Badminton club.

So, the largest possible number of students who are neither in Swimming club nor Badminton club is $100 - 86 = \mathbf{14}$.

Logic & Non-Routine Problems

3. In a 4-digit number ABCD, the digits A, B, C and D are in decreasing order from left to right. What is the largest possible difference AC − BD of the 2-digit numbers BD and AC?

Solution: To obtain the largest possible difference, A must be maximised while B must be minimised. Therefore, A is 9, while BCD are 3 consecutive numbers in descending order. To minimise B, D must be 0, hence C is 1 and B is 2. The largest possible difference = 91 – 20 = 71

Logic & Non-Routine Problems

4. Mr. Tan placed 10 coins on a table, with all heads up. He then asked his son to flip exactly 3 coins in each round. What is the least number of rounds needed to ensure all the coins have tails up?

Solution:

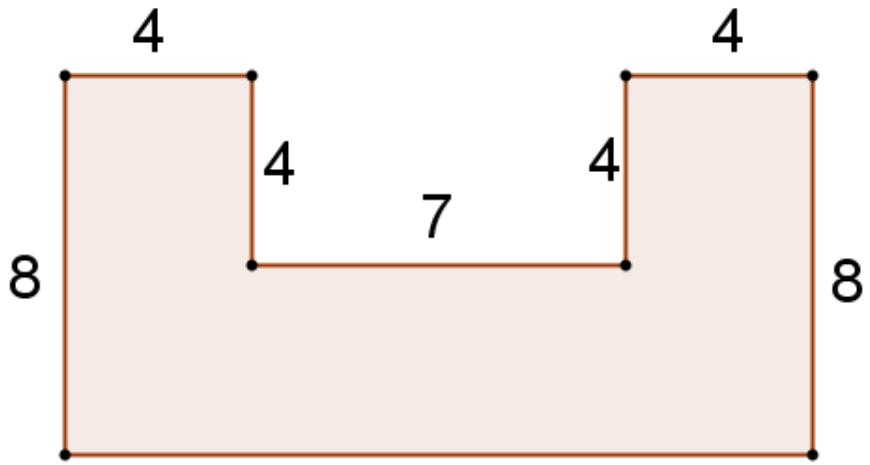
4 rounds: HHHHH HHHHH → HHHHH HHTTT

→ HHHTT HHHTT → TTTTT HHHTT → TTTTT TTTTT

Answer: 4

Geometric & Spatial Reasoning

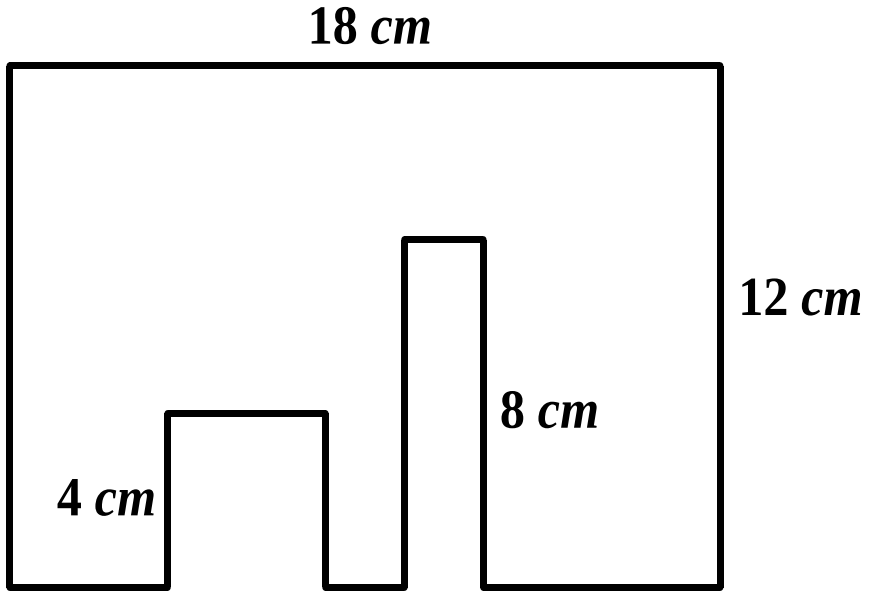
Example: What is the perimeter of the shape below?



Answer: $8 + 4 + 4 + 7 + 4 + 4 + 8 + (4 + 7 + 4) = \mathbf{54}$

Geometric & Spatial Reasoning

5. Find the perimeter of the shape below.

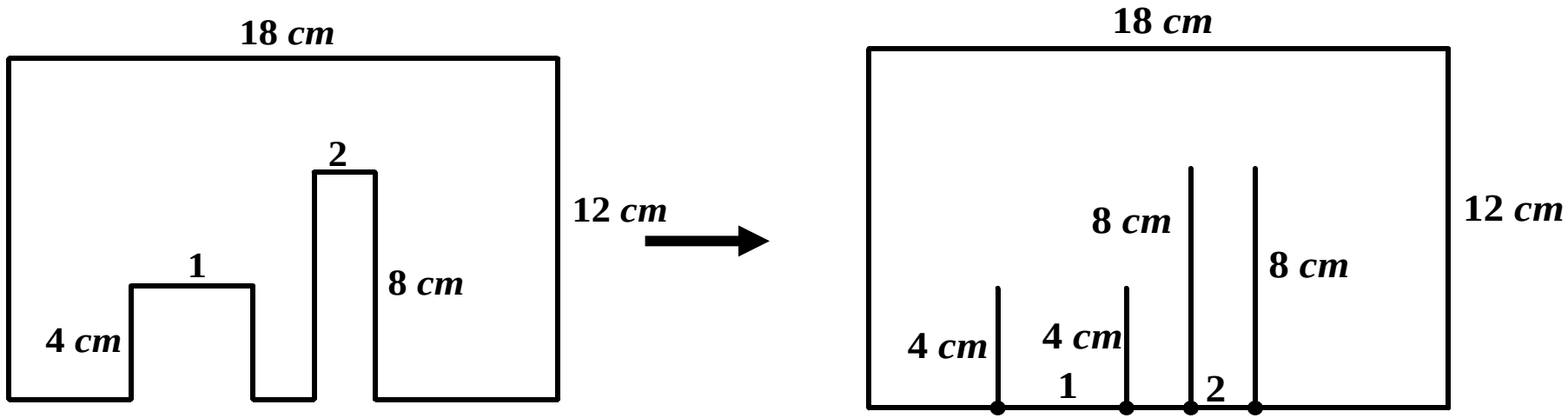


Geometric & Spatial Reasoning

Solution:

Let us label the side with ‘1’ and ‘2’ as shown on the figure below. By moving the labelled sides, we construct the new figure which is a rectangle. The perimeter of the figure is the sum of the perimeter of the rectangle and the length of the 4 sides.

The perimeter is $2 \times (18 + 12) + 4 + 4 + 8 + 8 = 84$ cm



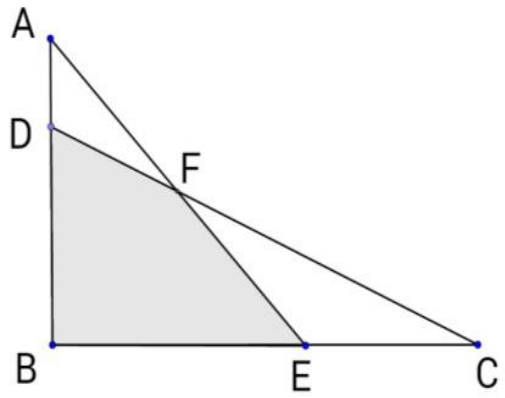
Answer: 84 cm

Geometric & Spatial Reasoning

6.

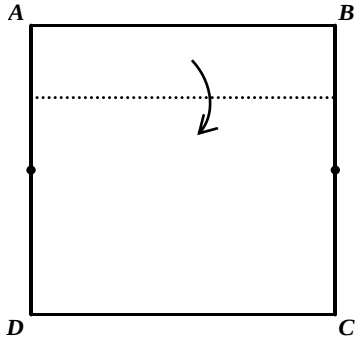
ΔABE and ΔDBC overlapped as shown below. Given that $\angle ABC = 90^0$, $AD = 2\ cm$, $DB = 6\ cm$, $BE = 8\ cm$ and $EC = 4\ cm$, find the area of the shaded region $DFEB$ in cm^2.

Answer: 28



Geometric & Spatial Reasoning

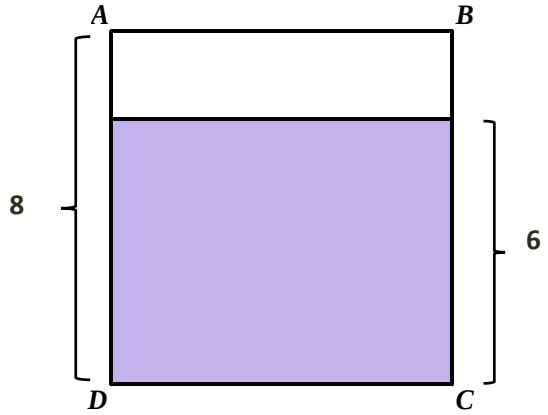
7. ABCD is a square piece of paper. Corner A is folded onto the midpoint of side AD. If the area of ABCD is 64 cm^2, what is the area of the new figure?



Solution: If the area of ABCD is 64, then the length of the side of ABCD is 8 cm.

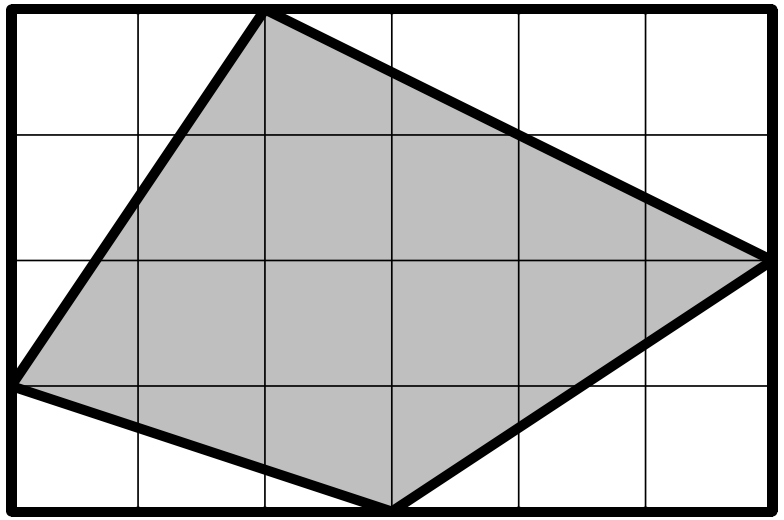
Thus, the area of the shaded region is $8 \times 6 = 48\ cm^2$

Answer: 48



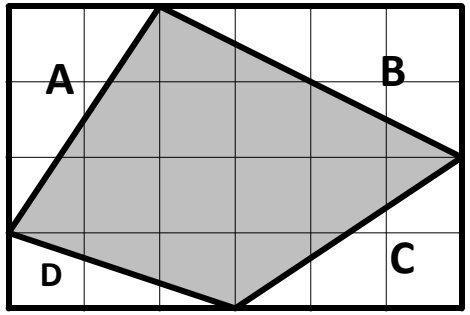
Geometric & Spatial Reasoning

8. Find the area of the shaded region if the side of each square grid is 1 cm.



Geometric & Spatial Reasoning

Solution:



Let us denote four unshaded regions as A, B, C and D.

Area of A is half area of the rectangle 3 by $2 = 3 \times 2 \div 2 = 3\ cm^2$

Area of B is $4 \times 2 \div 2 = 4\ cm^2$

Area of C is $3 \times 2 \div 2 = 3\ cm^2$

Area of D is $3 \times 1 \div 2 = 1.5\ cm^2$

Area of the rectangle is $6 \times 4 = 24\ cm^2$

Area of the shaded region is

$$Area\ of\ rectangle - Area\ of\ (A + B + C + D) = 24 - 11.5 = 12.5\ \ cm^2.$$

Geometric & Spatial Reasoning

9. Nick built a cube using gray and white cubes. The cubes with the same coluor do not have a common face. Which statement describes the number of used cubes?

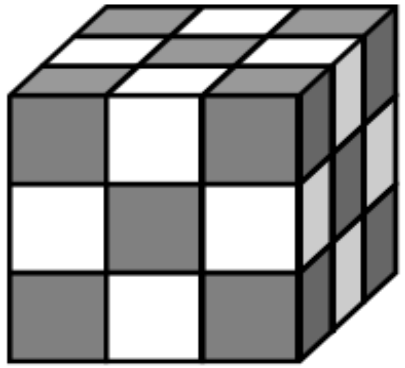
(**A**) one gray cube more than white cubes

(**B**) one white cube more than gray cubes

(**C**) the same number of gray and white cubes

(**D**) two white cubes more than gray cubes

(**E**) two gray cubes more than white cubes



Answer: A

Top layer: 5 gray, 4 white

Middle layer: 4 gray, 5 white

Bottom layer: 5 gray, 4 white

Total gray $= 5 + 4 + 5 = 14$, total white $= 4 + 5 + 4 = 13$

Arithmetic and Number Concepts

Order of operations: BDMAS

Do things in Brackets (**B**) First: $6 \times (5 + 3) = 6 \times 8 = 48$

Multiply or Divide (**DM**) before you Add or Subtract (**AS**): $2 + 5 \times 3 = 2 + 15 = 17$

Otherwise just go left to right: $30 \div 5 \times 3 = 6 \times 3 = 18$

Arithmetic and Number Concepts

10. Find the value of $2018 + 2018 - 2018 \times 2018 \div 2018$.

Solution:

Following the BDMAS rule,

$$2018 + 2018 - 2018 \times 2018 \div 2018 = 2018 + 2018 - (2018 \times 2018 \div 2018) = 2018 + 2018 - 2018 = 2018$$

Arithmetic and Number Concepts

11. If the sum of three times a prime number and twice another prime is 40, what is the difference of these two prime numbers?

Ans: 15

$Odd \times Odd = Odd$ $Odd \times Even = Even$ $Even + Even = Odd$ $Even + Odd = Odd$

$3 \times prime + 2 \times prime = 40\ (?) + even = even$

$3 \times prime$ must be an even number, so the first prime number is 2.

$3 \times 2 + 2 \times prime = 40$.Second prime number is $(40 - 6) \div 2 = 17$.

The difference between two prime numbers $= 17 - 2 = 15$

Arithmetic and Number Concepts

Example: If 6 workers can harvest a field in 18 hours, how many workers would it have taken to do it in 3 hours?

Solution:

6 workers can harvest a field in 18 hours

1 worker can harvest a field in $18 \times 6 = 108$ hours

$108 \div 3 = \mathbf{36}$ **workers**

Arithmetic and Number Concepts

12. Working together, 8 students can solve 20 problems in 10 minutes. At the same rate, how many students are needed in order to solve 30 problems in 5 minutes?

Solution:

Idea: The shorter the time, less problem can be solved.

More people, more problems can be solved.

Number of human/machine	Work done	Time
8	20	10
8	$20 \div 2 = 10$	$10 \div 2 = 5$
$8 \times 3 =$**24**	$10 \times 3 = 30$	5

Answer: **24 students**

Arithmetic and Number Concepts

13. A, B and C will work on a project. It would take 8 days if A and B work together and 6 days if B and C work together. If all of them work together, it would take only 4 days. How many days would it take if B work alone? (Assume each has constant work rate)

Ans: 24

$Work\ rate\ of\ A + B = \frac{1}{8}\ Work\ rate\ of\ B + C = \frac{1}{6}\ Work\ rate\ of\ A + B + C = \frac{1}{4}$

$Work\ rate\ of\ A = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}\ Work\ rate\ of\ C = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}\ Work\ rate\ of\ B = \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

It will take 24 days for B to complete the job.

Arithmetic and Number Concepts

14. Mickey and Minnie had some durians in the ratio 5:14. After Mickey gave Minnie some durians, the ratio of the number of durians Mickey had to that of Minnie was 13:63. Express the number of durians that Mickey gave Minnie as a percentage of the number of durians he had at first.

Solution

Note that 5 + 14 = 19 and 17 + 59 = 76. Since the total number of durians remain unchanged after the transfer, we shall multiply by 4 to both sides of the ratio 5:14 to obtain 20: 56.

Hence, the required percentage is $\frac{20-13}{20} \times 100\% = 35\%$.

Arithmetic and Number Concepts

15. There are 15 students in a class and each student has a whole number from 1 to 15. They received different numbers.
Student #1 wrote a whole number on the board (not the number he got).
Student #2 said, "This number is divisible by 2."
Student #3 said, "This number is divisible by my number (which is 3)."
Student #4 said, "This number is divisible by my number (which is 4)."
And so on until…
Student #15 said, "This number is divisible by my number (which is 15)."

Student #1 verified what the other 14 students said and he found that all of them are correct except for two students whose numbers are consecutive. What is the sum of these two consecutive numbers?

Ans: 17

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Assume that the number is divisible by 15. This means that the number is also divisible by 3 and 5.
Assume that the number is divisible by 14. This means that the number is also divisible by 2 and 7.
Assume that the number is divisible by 12. This means that the number is also divisible by 2, 3, 4 and 6.

Since the number is divisible by 2 and 5, the number must be divisible by 10.
From our assumption above, the number can be divisible by 1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15

The two only 2 consecutive numbers left are 8 and 9. The sum is 17.

Counting Techniques

Example: Seven kids are in a birthday party. Each of them shakes his/her hand only once with every other kids. What is the total number of handshakes in this birthday party?

Solution:

1st kid will handshake with the remaining 6.

2nd kid will handshake with the remaining 5.

3rd kid will handshake with the remaining 4.

4th kid will handshake with the remaining 3.

5th kid will handshake with the remaining 2.

6th kid will handshake with the remaining 1.

7th kid had shaken hands with the others.

Total = $6 + 5 + 4 + 3 + 2 + 1 = \mathbf{21}$

Counting Techniques

16. There are 21 points on a circle. How many lines can be formed by connecting any two of these points?

Answer = $20 + 19 + 18 + \ \ldots + 1 = 210$

Counting Techniques

17. How many even 2-digit numbers have an odd number as the sum of their digits?

Solution: Consider the ones digit first.

The number is even, so the ones digit is even: 0, 2, 4, 6, and 8 are the five possibilities. The sum of the digits is odd, so the tens digit is odd: 1, 3, 5, 7, and 9 are the five possibilities. For each possible ones digit, there are five possible tens digits. Altogether, there are 5×5 or 25 even two-digit numbers that have an odd number as the sum of their digits.

Answer: **25**

Counting Techniques

18. Consider a 2–digit number. If this number's ones and tens digits are switched, then the new 2–digit number is at least 3 times the original number. How many such 2–digit numbers are there?

Solution:

By guess and check, the following are the only 6 possible numbers that satisfy the criteria: 15, 16, 17, 18, 19, 29

Answer: 6